Note Title 3/28/2019

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\boxed{Y(\omega) = H(\omega)\, X(\omega)}$$

Frequency - Domain Representation

$$|Y(\omega)| = |H(\omega)|\,|X(\omega)|$$

$$\angle Y(\omega) = \angle H(\omega) + \angle X(\omega)$$

$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$

Response to Sinusoidal Input

$$x(t) = A\cos(\omega_0 t + \theta) \qquad -\infty < t < \infty$$

$$X(\omega) = A\pi\left[e^{-j\theta}\delta(\omega+\omega_0) + e^{j\theta}\delta(\omega-\omega_0)\right]$$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(\omega)$

$A \leftrightarrow 2\pi A\,\delta(\omega)$

$$Y(\omega) = H(\omega)X(\omega)$$

$$Y(\omega) = A\pi H(\omega)\left[e^{-j\theta}\delta(\omega+\omega_0) + e^{j\theta}\delta(\omega-\omega_0)\right]$$

$$= A\pi\left[e^{-j\theta}\delta(\omega+\omega_0)H(\omega) + e^{j\theta}\delta(\omega-\omega_0)H(\omega)\right]$$

$$\delta(\omega+\omega_0)H(\omega) = H(-\omega_0)\,\delta(\omega+\omega_0)$$

$$\delta(\omega-\omega_0)H(\omega) = H(\omega_0)\,\delta(\omega-\omega_0)$$

$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{o.w.} \end{cases}$

$$Y(\omega) = A\pi\left[e^{j\theta} H(-\omega_0)\,\delta(\omega+\omega_0) + e^{j\theta} H(\omega_0)\,\delta(\omega-\omega_0)\right]$$

$$|H(-\omega_0)| = |H(\omega_0)|$$

$$\angle H(-\omega_0) = -\angle H(\omega_0)$$

$$H(-\omega_0) = |H(\omega_0)|\, e^{-j\angle H(\omega_0)}$$

$$H(\omega_0) = |H(\omega_0)|\, e^{j\angle H(\omega_0)}$$

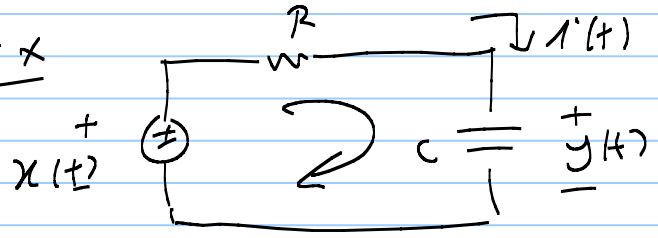
$$Y(\omega) = A\pi|H(\omega_0)|\left[e^{-j(\theta+\angle H(\omega_0))}\delta(\omega+\omega_0) + e^{j(\theta+\angle H(\omega_0))}\delta(\omega-\omega_0)\right]$$

$$\boxed{y(t) = A\,|H(\omega_0)|\cos\left(\omega_0 t + \theta + \angle H(\omega_0)\right)}$$

$x(t) = A\cos(\omega_0 t + \theta)$

↑ This is for LTI & sinusoidal input

Ex



① Find $H(\omega)$

② Find $y(t)$ if $x(t) = 10\cos(1000t + 20^\circ)$

(1) $-x(t) + R\,i(t) + y(t) = 0$ | $i(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt}$

$$-x(t) + RC\frac{dy(t)}{dt} + y(t) = 0$$

$$\boxed{\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)}$$

Use the derivative property

$$jwY(w) + \frac{1}{RC}Y(w) = \frac{1}{RC}X(w)$$

$$Y(w)\left[\frac{1}{RC} + jw\right] = \frac{1}{RC}X(w)$$

$$H(w) = \frac{Y(w)}{X(w)} = \frac{1/RC}{1/RC + jw}$$ ← Transfer function

To find $H(w)$ Assume all I.C.'s are zero

(2)

$$x(t) = 10\cos(1000t + 20^\circ)$$ (w_0 = 1000)

$$y(t) = 10\,|H(1000)|\cos(1000t + 20 + \angle H(1000))$$

$$H(1000) = \frac{1/RC}{1/RC + j1000}$$

$$y(t) = 10\left|\frac{1/RC}{1/RC + j1000}\right|\cos\left(1000t + 20 - \tan^{-1}(1000RC)\right)$$

If $1/RC = 1000 \rightarrow H(1000) = \frac{1}{\sqrt{2}}$; $\angle H(1000) = -\tan^{-1}(1)$

If $1/RC = 1 \rightarrow H(1000) \sim 0.001$; $\angle H(1000) = -\tan^{-1}(1000)$

If $1/RC = 10^{\infty} \rightarrow H(j\omega) \sim 1$; $\angle H(j\omega) \sim -90^{\circ}$

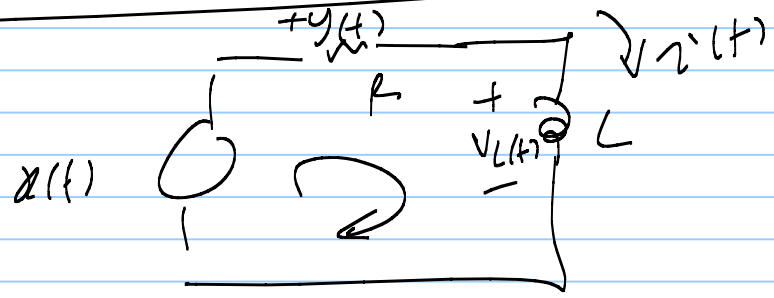
$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\pi t), -\infty < t < \infty$$

where $a_0 = 0.5$ and

$$a_k = \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right)$$

- $H(\omega) = \frac{1/RC}{j\omega + 1/RC}$

$$y(t) = a_0 |H(0)| \cos(0 + \angle H(0)) + \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right) \left| \frac{1/RC}{jk\pi + 1/RC} \right| \cos\left(k\pi t + \angle \frac{1/RC}{jk\pi + 1/RC}\right)$$



$$V_L(t) = L \frac{di(t)}{dt}$$

$$i(t) = \frac{y(t)}{R}$$

$$V_L(t) = \frac{L}{R} \frac{dy(t)}{dt}$$

$$-x(t) + y(t) + V_L(t) = 0$$

$$-x(t) + y(t) + \frac{L}{R} \frac{dy(t)}{dt} = 0$$

$$-X(\omega) + Y(\omega) + j\omega \frac{L}{R} Y(\omega) = 0$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{1}{1 + j\omega \frac{L}{R}} = \frac{R/L}{R/L + j\omega} = H(\omega)$$

$$H(\omega) = \frac{100}{100 + j\omega}$$

$y(t) = ?$

$$H(0) = 1$$

$$H(10) = \frac{100}{100 + j10}$$

$$H(1000) = \frac{100}{100 + j100}$$